Tonalness Equations

1. $$X(f) = 2\int_{-\infty}^{\infty} e^{-2\pi i t f} P(t)\, dt$$

2. $$Y_h(f,t) = 2R(hf)\int_{-\infty}^{\infty} \frac{e^{-\frac{(u-t)^2}{2\hat{\sigma}_h(f)^2}} e^{-2\pi i(u-t)hf} P(u)}{\sqrt{2\pi}\hat{\sigma}_h(f)}\, du$$

3. $$\hat{\sigma}_h(f) = \frac{1}{2\pi\sigma_h(f)}$$

4. $$\sigma_h(f) = \kappa\left(\frac{24.7}{h} + 0.107939f\right)$$

5. $$R(f) = \frac{12194^2 f^4}{\left(f^2+20.6^2\right)\left(f^2+12194^2\right)\sqrt{\left(f^2+107.7^2\right)\left(f^2+737.9^2\right)}}$$

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6. $$\tau(f,t) = \left| \sum_{h=1}^{\infty} W_h(f,t) \left| Y_h(f,t) \right| \exp\left(\frac{i\Phi_h(f,t)}{h} \right) \right|$$

7. $$W_h(f,t) = \begin{cases} \sqrt{1 - \sum_{k=2}^{\infty} W_k(f/k,t)^2 \frac{|Y_1(f/k,t)|}{|Y_1(f/k,t)| + \zeta}} & \text{if } h = 1 \\ \frac{1}{h}\left(1 - \frac{|Y_h(f,t)|}{\sum_{g=1}^{\infty} |Y_g(f,t)| + \delta}\right) & \text{otherwise} \end{cases}$$

8. $$\Phi_h(f,t) = \arg\left(Y_h\left(f,t_0\right)\right) + \int_{t_0}^{t} \Im\left(\frac{\frac{\partial Y_h(f,u)}{\partial u}}{Y_h(f,u)} \right) du$$

9. $$\psi_1(f,t) = \max_{|u-t|<\frac{T_c}{2}} \tau(f,u)$$

Tonalness Equations

10. $$\psi_2(f,t) = \sqrt{\frac{1}{f} \max_{|u-t|<\frac{T_c}{2}} \left| \frac{\partial}{\partial u} \tau(f,u)^2 \right|}$$

11. $$\eta(t) = \frac{\int_0^\infty \frac{\psi_2(f,t)^2}{\sigma_1(f)}\, df}{\int_0^\infty \frac{\psi_1(f,t)^2}{\sigma_1(f)}\, df + \delta}$$

12. $$\mu(t) = \sqrt{\int_0^\infty \frac{\psi_1(f,t)^2}{\sigma_1(f)}\, df} + \delta$$

13. $$\Omega\left(t_1,t_2\right) = \frac{1}{2}\int_0^\infty \frac{1}{\sigma_1(f)} \left(\frac{\psi_1\left(f,t_1\right)}{\mu\left(t_1\right)} - \frac{\psi_1\left(f,t_2\right)}{\mu\left(t_2\right)} \right)^2 df$$

14. $$b(s) = b_0 2^{\frac{1}{12}(s-s_0)}$$

Tonalness Equations

15. $$P(t) = \sum_{i=1}^{n} a_i \cos\left(2\pi b_i t + c_i\right)$$

16. $$\tau(f,t) = \left| \sum_{h=1}^{\infty} \sum_{i=1}^{n} W_h(f,t) U_{h,i}(f) e^{\frac{i\left(2\pi t b_i + c_i\right)}{h}} \right|$$

17. $$U_{h,i}(f) = l_i e^{-\frac{(hf - b_i)^2}{2\sigma_h(f)^2}}$$

18. $$l_i = a_i R\left(b_i\right)$$

19. $$W_h(f,t) = \begin{cases} \sqrt{1 - \sum_{k=2}^{\infty} W_k(f/k,t)^2 \frac{\sum_{i=1}^{n} U_{1,i}(f/k)}{\sum_{i=1}^{n} U_{1,i}(f/k) + \zeta}} & \text{if } h = 1 \\ \frac{1}{h}\left(1 - \frac{\sum_{i=1}^{n} U_{h,i}(f)}{\sum_{i=1}^{n} \sum_{g=1}^{\infty} U_{g,i}(f) + \delta}\right) & \text{otherwise} \end{cases}$$

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20. $$\psi_1(f,t) = \sum_{h=1}^{\infty} \sum_{i=1}^{n} W_h(f,t) U_{h,i}(f)$$

21. $$\psi_2(f,t) = \sqrt{\frac{2\pi}{f} \sum_{h_1=1}^{\infty} \sum_{h_2=1}^{\infty} \sum_{i_1=1}^{n} \sum_{i_2=1}^{n} W_{h_1}(f,t) W_{h_2}(f,t) U_{h_1,i_1}(f) U_{h_2,i_2}(f) \left| \frac{b_{i_2}}{h_2} - \frac{b_{i_1}}{h_1} \right|}$$

22. $$\eta(t) = \frac{2\pi \sum_{h_1=1}^{\infty} \sum_{h_2=1}^{\infty} \sum_{i_1=1}^{n} \sum_{i_2=1}^{n} \frac{W_{h_1}\left(\frac{b_{i_1}}{h_1}, t\right) W_{h_2}\left(\frac{b_{i_2}}{h_2}, t\right) \gamma_{h_1,h_2,i_1,i_2} \left| \frac{b_{i_2}}{h_2} - \frac{b_{i_1}}{h_1} \right|}{\phi_{h_1,h_2,i_1,i_2}}}{\sum_{h_1=1}^{\infty} \sum_{h_2=1}^{\infty} \sum_{i_1=1}^{n} \sum_{i_2=1}^{n} W_{h_1}\left(\frac{b_{i_1}}{h_1}, t\right) W_{h_2}\left(\frac{b_{i_2}}{h_2}, t\right) \gamma_{h_1,h_2,i_1,i_2}}$$

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23.
$$\gamma_{h_1,h_2,i_1,i_2} = \frac{l_{i_1} l_{i_2} \sigma_{h_1} \left(\frac{b_{i_1}}{h_1}\right) \sigma_{h_2} \left(\frac{b_{i_2}}{h_2}\right) \exp\left(-\frac{\left(b_{i_2} h_1 - b_{i_1} h_2\right)^2}{2\left(h_2^2 \sigma_{h_1}\left(\frac{b_{i_1}}{h_1}\right)^2 + h_1^2 \sigma_{h_2}\left(\frac{b_{i_2}}{h_2}\right)^2\right)}\right)}{\sigma_1\left(\phi_{h_1,h_2,i_1,i_2}\right)\sqrt{h_2^2 \sigma_{h_1}\left(\frac{b_{i_1}}{h_1}\right)^2 + h_1^2 \sigma_{h_2}\left(\frac{b_{i_2}}{h_2}\right)^2}}$$

24.
$$\phi_{h_1,h_2,i_1,i_2} = \frac{b_{i_2} h_2 + b_{i_1} h_1}{h_2^2 + h_1^2}$$

25.
$$\psi_1\left(f, t_x\right) = \sum_{h=1}^{\infty} \sum_{i=1}^{n_x} W_h\left(f, t_x\right) U_{h,i,x}(f)$$

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26.
$$W_h(f,t_x)=\begin{cases}\sqrt{1-\sum_{k=2}^{\infty}W_k(f/k,t_x)^2\frac{\sum_{i=1}^{n_x}U_{1,i,x}(f/k)}{\sum_{i=1}^{n_x}U_{1,i,x}(f/k)+\zeta}} & \text{if } h=1\\ \frac{1}{h}\left(1-\frac{\sum_{i=1}^{n_x}U_{h,i,x}(f)}{\sum_{i=1}^{n_x}\sum_{g=1}^{\infty}U_{g,i,x}(f)+\delta}\right) & \text{otherwise}\end{cases}$$

27.
$$U_{h,i,x}(f)=l_{i,x}e^{-\frac{\left(hf-b_{i,x}\right)^2}{2\sigma_h(f)^2}}$$

28.
$$\mu\left(t_x\right)=\sqrt{\sum_{h_1=1}^{\infty}\sum_{h_2=1}^{\infty}\sum_{i_1=1}^{n_x}\sum_{i_2=1}^{n_x}W_{h_1}\left(\frac{b_{i_1,x}}{h_1},t_x\right)W_{h_2}\left(\frac{b_{i_2,x}}{h_2},t_x\right)\gamma_{h_1,h_2,i_1,i_2}(x,x)}$$

29.
$$\Omega\left(t_1,t_2\right)=1-\frac{1}{\mu\left(t_1\right)\mu\left(t_2\right)}\sum_{h_1=1}^{\infty}\sum_{h_2=1}^{\infty}\sum_{i_1=1}^{n_1}\sum_{i_2=1}^{n_2}W_{h_1}\left(\frac{b_{i_1,1}}{h_1},t_1\right)W_{h_2}\left(\frac{b_{i_2,2}}{h_2},t_2\right)\gamma_{h_1,h_2,i_1,i_2}(1,2)$$

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30.
$$\gamma_{h_1,h_2,i_1,i_2}(x,y) = \frac{l_{i_1,x} l_{i_2,y} \sigma_{h_1}\left(\frac{b_{i_1,x}}{h_1}\right) \sigma_{h_2}\left(\frac{b_{i_2,y}}{h_2}\right) \exp\left(-\frac{\left(h_2 b_{i_1,x} - h_1 b_{i_2,y}\right)^2}{2\left(h_2^2 \sigma_{h_1}\left(\frac{b_{i_1,x}}{h_1}\right)^2 + h_1^2 \sigma_{h_2}\left(\frac{b_{i_2,y}}{h_2}\right)^2\right)}\right)}{\sigma_1\left(\phi_{h_1,h_2,i_1,i_2}(x,y)\right)\sqrt{h_2^2 \sigma_{h_1}\left(\frac{b_{i_1,x}}{h_1}\right)^2 + h_1^2 \sigma_{h_2}\left(\frac{b_{i_2,y}}{h_2}\right)^2}}$$

31.
$$\phi_{h_1,h_2,i_1,i_2}(x,y) = \frac{h_2 b_{i_2,y} + h_1 b_{i_1,x}}{h_2^2 + h_1^2}$$