Tonalness Equations

1. $$X(f) = 2\int_{-\infty}^{\infty} e^{-2\pi itf} P(t)\, dt$$

2. $$Y_h(f,t) = \frac{2R(hf)}{h}\int_{-\infty}^{\infty} \frac{e^{-\frac{(u-t)^2}{2\hat{\sigma}_h(f)^2}} e^{-2\pi i(u-t)hf} P(u)}{\sqrt{2\pi}\hat{\sigma}_h(f)}\, du$$

3. $$\hat{\sigma}_h(f) = \frac{1}{2\pi\sigma_h(f)}$$

4. $$\sigma_h(f) = \kappa\left(\frac{24.7}{h} + 0.107939f\right)$$

5. $$R(f) = \frac{12194^2 f^4}{\left(f^2+20.6^2\right)\left(f^2+12194^2\right)\sqrt{\left(f^2+107.7^2\right)\left(f^2+737.9^2\right)}}$$

Tonalness Equations

6. $$\tau(f,t) = \left| \sum_{h=1}^{\infty} W_h(f,t) \left| Y_h(f,t) \right| \exp\left(\frac{i\Phi_h(f,t)}{h} \right) \right|$$

7. $$W_h(f,t) = \mathsf{If}\left[h = 1,1,1 - \frac{\left| Y_h(f,t) \right|}{\sum_{g=1}^{\infty} \left| Y_g(f,t) \right|} \right]$$

8. $$\Phi_h(f,t) = \arg\left(Y_h\left(f, t_0\right) \right) + \int_{t_0}^{t} \Im\left(\frac{\frac{\partial Y_h(f,u)}{\partial u}}{Y_h(f,u)} \right) du$$

9. $$P(t) = \sum_{i=1}^{n} a_i \cos\left(2\pi b_i t + c_i\right)$$

Tonalness Equations

10. $$\tau(f,t) = \left| \sum_{h=1}^{\infty} \sum_{i=1}^{n} W_h(f,t) U_{h,i}(f) e^{\frac{i(2\pi t b_i + c_i)}{h}} \right|$$

11. $$U_{h,i}(f) = \frac{l_i}{h} e^{-\frac{(hf - b_i)^2}{2\sigma_h(f)^2}}$$

12. $$l_i = a_i R\left(b_i\right)$$

13. $$W_h(f,t) = \text{If}\left[h = 1,1,1 - \frac{\sum_{i=1}^{n} U_{h,i}(f)}{\sum_{i=1}^{n} \sum_{g=1}^{\infty} U_{g,i}(f)}\right]$$

14. $$\psi_1(f,t) = \sqrt{\text{Maximize}\left[\tau(f,u)^2, u \in |u - t| < \frac{T_c}{2}\right]}$$

Tonalness Equations

15. $$\psi_2(f,t) = \sqrt{\frac{1}{f}\text{Maximize}\left[\frac{\partial\tau(f,u)^2}{\partial u}, u \in |u-t| < \frac{T_c}{2}\right]}$$

16. $$\psi_1(f,t) = \sum_{h=1}^{\infty}\sum_{i=1}^{n} W_h(f,t)U_{h,i}(f)$$

17. $$\psi_2(f,t) = \sqrt{\frac{2\pi}{f}\sum_{h_1=1}^{\infty}\sum_{h_2=1}^{\infty}\sum_{i_1=1}^{n}\sum_{i_2=1}^{n} W_{h_1}(f,t)W_{h_2}(f,t)U_{h_1,i_1}(f)U_{h_2,i_2}(f)\left|\frac{b_{i_2}}{h_2}-\frac{b_{i_1}}{h_1}\right|}$$

18. $$\eta(t) = \frac{\int_{-\infty}^{\infty}\psi_2(f,t)^2\,df}{\int_{-\infty}^{\infty}\psi_1(f,t)^2\,df}$$

19. $$\eta(t) = \frac{2\pi\sum_{h_1=1}^{\infty}\sum_{h_2=1}^{\infty}\sum_{i_1=1}^{n}\sum_{i_2=1}^{n}\frac{W_{h_1}(f,t)W_{h_2}(f,t)\gamma_{h_1,h_2,i_1,i_2}}{\phi_{h_1,h_2,i_1,i_2}}\left|\frac{b_{i_2}}{h_2}-\frac{b_{i_1}}{h_1}\right|}{\sum_{h_1=1}^{\infty}\sum_{h_2=1}^{\infty}\sum_{i_1=1}^{n}\sum_{i_2=1}^{n} W_{h_1}(f,t)W_{h_2}(f,t)\gamma_{h_1,h_2,i_1,i_2}}$$

Tonalness Equations

20. $$\phi_{h_1,h_2,i_1,i_2} = \frac{b_{i_1} h_1 + b_{i_2} h_2}{h_1^2 + h_2^2}$$

21. $$\gamma_{h_1,h_2,i_1,i_2} = \frac{l_{i_1} l_{i_2} \sigma_{h_1}\left(\phi_{h_1,h_2,i_1,i_2}\right) \sigma_{h_2}\left(\phi_{h_1,h_2,i_1,i_2}\right)}{h_1 h_2 \sqrt{h_1^2 \sigma_{h_1}\left(\phi_{h_1,h_2,i_1,i_2}\right)^2 + h_2^2 \sigma_{h_2}\left(\phi_{h_1,h_2,i_1,i_2}\right)^2}} \exp\left(-\frac{\left(b_{i_2} h_1 - b_{i_1} h_2\right)^2}{2\left(h_2^2 \sigma_{h_1}\left(\phi_{h_1,h_2,i_1,i_2}\right)^2 + h_1^2 \sigma_{h_2}\left(\phi_{h_1,h_2,i_1,i_2}\right)^2\right)}\right)$$

22. $$\Omega\left(t_1, t_2\right) = \frac{\sqrt{\int_{-\infty}^{\infty}\left(\psi_1\left(f, t_1\right) - \psi_1\left(f, t_2\right)\right)^2 df}}{\sqrt{\int_{-\infty}^{\infty}\left(\psi_1\left(f, t_1\right) + \psi_1\left(f, t_2\right)\right)^2 df}}$$

23. $$\psi_1\left(f, t_x\right) = \sum_{h=1}^{\infty} \sum_{i=1}^{n_x} W_h\left(f, t_x\right) U_{h,i,x}(f)$$

Tonalness Equations

24. $$W_h\left(f, t_x\right) = \text{If}\left[h = 1{,}1{,}1 - \frac{\sum_{i=1}^{n_x} U_{h,i,x}(f)}{\sum_{i=1}^{n_x}\sum_{g=1}^{\infty} U_{g,i,x}(f)}\right]$$

25. $$U_{h,i,x}(f) = \frac{l_{i,x}}{h} e^{-\frac{\left(hf - b_{i,x}\right)^2}{2\sigma_h(f)^2}}$$

26. $$\Omega\left(t_t, t_2\right) = \sqrt{\frac{G(1{,}1) - 2G(1{,}2) + G(2{,}2)}{G(1{,}1) + 2G(1{,}2) + G(2{,}2)}}$$

27. $$G(x,y) = \sum_{h_1=1}^{\infty}\sum_{h_2=1}^{\infty}\sum_{i_1=1}^{n_x}\sum_{i_2=1}^{n_y} W_{h_1}\left(\phi_{h_1,h_2,i_1,i_2}(x,y), t_x\right) W_{h_2}\left(\phi_{h_1,h_2,i_1,i_2}(x,y), t_y\right) \gamma_{h_1,h_2,i_1,i_2}(x,y)$$

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28.

$$\gamma_{h_1,h_2,i_1,i_2}(x,y) = \frac{l_{i_1,x} l_{i_2,y} \sigma_{h_1}\left(\phi_{h_1,h_2,i_1,i_2}(x,y)\right) \sigma_{h_2}\left(\phi_{h_1,h_2,i_1,i_2}(x,y)\right)}{h_1 h_2 \sqrt{h_2^2 \sigma_{h_1}\left(\phi_{h_1,h_2,i_1,i_2}(x,y)\right)^2 + h_1^2 \sigma_{h_2}\left(\phi_{h_1,h_2,i_1,i_2}(x,y)\right)^2}} \exp\left(-\frac{\left(h_2 b_{i_1,x} - h_1 b_{i_2,y}\right)^2}{2\left(h_2^2 \sigma_{h_1}\left(\phi_{h_1,h_2,i_1,i_2}(x,y)\right)^2 + h_1^2 \sigma_{h_2}\left(\phi_{h_1,h_2,i_1,i_2}(x,y)\right)^2\right)}\right)$$

29.

$$\phi_{h_1,h_2,i_1,i_2}(x,y) = \frac{b_{i_1,x} h_1 + b_{i_2,y} h_2}{h_1^2 + h_2^2}$$

30.

$$b(s) = b_0 2^{\frac{1}{12}(s - s_0)}$$