

Tonalness Equations

1.
$$\tau(f, t) = 2 \sum_{h=1}^{\infty} \frac{1}{h} \int_{-\infty}^{\infty} \frac{e^{-\frac{(u-t/h)^2}{2\sigma_t(f)^2}}}{\sqrt{2\pi}\sigma_t(f)} e^{-2\pi i u h f} P(u) du$$

2.
$$\sigma_t(f) = \frac{1}{2\pi\sigma(f)}$$

3.
$$\sigma(f) = \kappa (d_1 + d_2 f)$$

4.
$$\{d_1, d_2\} = \{24.7, 0.107939\}$$

5.
$$P(t) = \sum_{i=1}^n a_i \cos(2\pi b_i t + c_i)$$

6.
$$\tau(f, t) = \sum_{h=1}^{\infty} \sum_{i=1}^n U_{h,i}(f) \left(\cos \left(2\pi f t - \frac{2\pi t b_i}{h} - c_i \right) - i \sin \left(2\pi f t - \frac{2\pi t b_i}{h} - c_i \right) \right)$$

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7.
$$U_{h,i}(f) = \frac{a_i}{h} e^{-\frac{(hf - b_i)^2}{2\sigma(f)^2}}$$

8.
$$\psi_1(f, t) = \text{Maximize} \left[|\tau(f, u)|^2, u \in |u - t| < \frac{\sigma_t(0)}{2} \right]$$

9.
$$\psi_2(f, t) = \frac{1}{f} \text{Maximize} \left[\frac{\partial |\tau(f, u)|^2}{\partial u}, u \in |u - t| < \frac{\sigma_t(0)}{2} \right]$$

10.
$$\psi_1(f, t) = \left(\sum_{h=1}^{\infty} \sum_{i=1}^n U_{h,i}(f) \right)^2$$

11.
$$\psi_2(f, t) = \frac{2\pi}{f} \sum_{g=1}^{\infty} \sum_{h=1}^{\infty} \sum_{i=1}^n \sum_{j=1}^n U_{g,i}(f) U_{h,j}(f) \left| \frac{b_j}{h} - \frac{b_i}{g} \right|$$

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12.
$$\eta(t) = \frac{\int_{-\infty}^{\infty} \psi_2(f, t) df}{\int_{-\infty}^{\infty} \psi_1(f, t) df}$$

13.
$$\eta(t) = \frac{2\pi \sum_{g=1}^{\infty} \sum_{h=1}^{\infty} \sum_{i=1}^n \sum_{j=1}^n \frac{\gamma_{g,h,i,j} \left| \frac{b_j}{h} - \frac{b_i}{g} \right|}{\phi_{g,h,i,j}}}{\sum_{g=1}^{\infty} \sum_{h=1}^{\infty} \sum_{i=1}^n \sum_{j=1}^n \gamma_{g,h,i,j}}$$

14.
$$\phi_{g,h,i,j} = \frac{gb_i + hb_j}{g^2 + h^2}$$

15.
$$\gamma_{g,h,i,j} = \frac{a_i a_j \sigma(\phi_{g,h,i,j}) \sqrt{2\pi}}{gh \sqrt{g^2 + h^2}} \exp \left(-\frac{(gb_j - hb_i)^2}{2(g^2 + h^2) \sigma(\phi_{g,h,i,j})^2} \right)$$

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16.
$$\Omega(t_1, t_2) = \sqrt{\frac{\int_{-\infty}^{\infty} (\psi_1(f, t_1) - \psi_1(f, t_2))^2 df}{\int_{-\infty}^{\infty} (\psi_1(f, t_1) + \psi_1(f, t_2))^2 df}}$$

17.
$$b(s) = b_0 2^{\frac{1}{12}(s - s_0)}$$