

Tonality Equations

1.
$$X(f, t) = 2 \int_{-\infty}^{\infty} e^{-\frac{(u-t)^2}{2\sigma_t^2}} e^{-2\pi i u f} P(u) du$$

2.
$$Y(h, f, t) = \frac{\sigma(f)}{\sigma_f(f)} \int_{-\infty}^{\infty} e^{-\frac{(v-hf)^2}{2\sigma_f(f)^2}} X\left(v, \frac{t}{h}\right) dv$$

3.
$$\tau(f, t) = \sum_{k=1}^m \frac{1}{\sqrt{g_k h_k}} \Re\left(Y(g_k, f, t)\right) \Re\left(Y(h_k, f, t)\right) + \Im\left(Y(g_k, f, t)\right) \Im\left(Y(h_k, f, t)\right)$$

4.
$$\sigma(f) = \kappa (d_1 + d_2 f)$$

5.
$$\{d_1, d_2\} = \{24.7, 0.107939\}$$

6.
$$\sigma_t = \frac{1}{2\pi\kappa d_1}$$

7.
$$\sigma(f)^2 = \left(\frac{1}{2\pi\sigma_t}\right)^2 + \sigma_f(f)^2$$

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8. $\vec{F}_r = \{q_1, q_2, \dots, q_m\}$

9. $q_k = \frac{h_k}{g_k}$

10. $q_k < q_{k+1}$

11. $g_k \in \mathbb{Z}_{>0}$

12. $h_k \in \mathbb{Z}_{>0}$

13. $1 \leq g_k h_k \leq r$

14. $\gcd(g_k, h_k) = 1$

15. $P(t) = \sum_{i=1}^n a_i \cos(2\pi b_i t + c_i)$

16. $Y(h, f, t) = \sum_{i=1}^n U_i(h, f) \left(\cos \left(2f\pi t - \frac{2\pi t b_i}{h} - c_i \right) - i \sin \left(2f\pi t - \frac{2\pi t b_i}{h} - c_i \right) \right)$

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17.
$$U_i(h, f) = a_i e^{-\frac{(hf - b_i)^2}{2\sigma(f)^2}}$$

18.
$$\tau(f, t) = \sum_{i=1}^n \sum_{j=1}^n \sum_{k=1}^m \frac{U_i(g_k, f) U_j(h_k, f)}{\sqrt{g_k h_k}} \cos \left(2\pi t \left(\frac{b_j}{h_k} - \frac{b_i}{g_k} \right) + c_j - c_i \right)$$

19.
$$\psi_1(f, t) = \text{Maximize} \left[|\tau(f, u)|, u \in |u - t| < \frac{\sigma_t}{2} \right]$$

20.
$$\psi_2(f, t) = \frac{1}{f} \text{Maximize} \left[\frac{\partial |\tau(f, u)|}{\partial u}, u \in |u - t| < \frac{\sigma_t}{2} \right]$$

21.
$$\psi_1(f, t) = \sum_{i=1}^n \sum_{j=1}^n \sum_{k=1}^m \frac{U_i(g_k, f) U_j(h_k, f)}{\sqrt{g_k h_k}}$$

22.
$$\psi_2(f, t) = \frac{2\pi}{f} \sum_{i=1}^n \sum_{j=1}^n \sum_{k=1}^m \frac{U_i(g_k, f) U_j(h_k, f)}{\sqrt{g_k h_k}} \left| \frac{b_j}{h_k} - \frac{b_i}{g_k} \right|$$

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23.
$$\eta(t) = \frac{\int_{-\infty}^{\infty} \psi_2(f, t) df}{\int_{-\infty}^{\infty} \psi_1(f, t) df}$$

24.
$$\eta(t) = 2\pi \frac{\sum_{i=1}^n \sum_{j=1}^n \sum_{k=1}^m \frac{\gamma_{i,j,k} \left| \frac{b_j}{h_k} - \frac{b_i}{g_k} \right|}{\sqrt{g_k h_k} \phi_{i,j,k}}}{\sum_{i=1}^n \sum_{j=1}^n \sum_{k=1}^m \frac{\gamma_{i,j,k}}{\sqrt{g_k h_k}}}$$

25.
$$\phi_{i,j,k} = \frac{b_i g_k + b_j h_k}{g_k^2 + h_k^2}$$

26.
$$\gamma_{i,j,k} = \frac{a_i a_j \sigma(\phi_{i,j,k}) \sqrt{2\pi}}{\sqrt{g_k^2 + h_k^2}} \exp \left(-\frac{(b_j g_k - b_i h_k)^2}{2 (g_k^2 + h_k^2) \sigma(\phi_{i,j,k})^2} \right)$$

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27.
$$\Omega(t_1, t_2) = \sqrt{\frac{\int_{-\infty}^{\infty} (\psi_1(f, t_1) - \psi_1(f, t_2))^2 df}{\int_{-\infty}^{\infty} (\psi_1(f, t_1) + \psi_1(f, t_2))^2 df}}$$

28.
$$b(s) = b_0 2^{\frac{1}{12}(s - s_0)}$$