Tonalness Equations

1. $F(f) = \int_{-\infty}^{\infty} P(t) e^{-2\pi i t f}\, dt$

2. $G(f,t) = \gamma(f) \int_{-\infty}^{\infty} \alpha(u,t) P(u) e^{-2\pi i u f}\, du$

3. $\alpha(u,t) = e^{-\frac{(u-t)^2}{2\sigma^2}}$

4. $\tau(f,t) = \frac{2}{\pi\sigma^2} \sum_{g=1}^{\infty} \sum_{h=g}^{\infty} \frac{\text{If}[\gcd(g,h) = 1,1,0]}{\sqrt{gh}} G\left(gf, \frac{t}{g}\right) G\left(hf, \frac{t}{h}\right)^*$

5. $\tau_0(f,t) = \frac{2}{\pi\sigma^2} G(f,t) G(f,t)^*$

6. $P(t) = \sum_{i=1}^{n} a_i \cos\left(2\pi b_i t + c_i\right)$

7. $$G(f,t) = \sigma\sqrt{\frac{\pi}{2}}\sum_{i=1}^{n} U_i(f)\left(V_i(f,t) - iW_i(f,t)\right)$$

8. $$U_i(f) = a_i e^{-2\pi^2\sigma^2(f-b_i)^2}$$

9. $$V_i(f,t) = \cos\left(2\pi t\left(f-b_i\right) - c_i\right)$$

10. $$W_i(f,t) = \sin\left(2\pi t\left(f-b_i\right) - c_i\right)$$

11. $$\tau(f,t) = \sum_{g=1}^{\infty}\sum_{h=g}^{\infty}\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{\text{If}[\gcd(g,h)=1,1,0]}{\sqrt{gh}}U_i(gf)U_j(hf)\left(X_{g,h,i,j}(t) - iY_{g,h,i,j}(t)\right)$$

12. $$X_{g,h,i,j}(t) = \cos\left(2\pi t\left(\frac{b_j}{h} - \frac{b_i}{g}\right) + c_j - c_i\right)$$

13. $$Y_{g,h,i,j}(t) = \sin\left(2\pi t\left(\frac{b_j}{h} - \frac{b_i}{g}\right) + c_j - c_i\right)$$

14. $s = \log_2\left(\frac{b}{b_0}\right)$

15. $b = b_0 2^{s/12}$

16. $\psi_1(f,t) = \sqrt{\int_{-\infty}^{\infty} \frac{\alpha(u,t)}{\sigma\sqrt{2\pi}} \tau(f,u)\tau(f,u)^* \, du}$

17. $\psi_2(f,t) = \sqrt{\int_{-\infty}^{\infty} \frac{\alpha(u,t)}{\sigma\sqrt{2\pi}} \frac{\partial\tau(f,u)}{\partial u} \left(\frac{\partial\tau(f,u)}{\partial u}\right)^* du}$

18. $\psi_1(f,t) = \sum_{g=1}^{\infty} \sum_{h=g}^{\infty} \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{\text{If}[\gcd(g,h) = 1,1,0]}{\sqrt{gh}} U_i(gf) U_j(hf)$

19. $\psi_2(f,t) = 2\pi \sum_{g=1}^{\infty} \sum_{h=g}^{\infty} \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{\text{If}[\gcd(g,h) = 1,1,0]}{\sqrt{gh}} \left|\frac{b_j}{h} - \frac{b_i}{g}\right| U_i(gf) U_j(hf)$

20. $$\eta_1(t) = \frac{1}{H(t)^2} \int_{-\infty}^{\infty} \left(\psi_1(f,t) - \psi_0(f,t)\right)^2 df$$

21. $$\eta_2(t) = \frac{1}{H(t)} \int_{-\infty}^{\infty} \psi_2(f,t)\, df$$

22. $$\psi_0(f,t) = \sqrt{\int_{-\infty}^{\infty} \frac{\alpha(u,t)}{\sigma\sqrt{2\pi}} \tau_0(f,u)\tau_0(f,u)^*\, du}$$

23. $$\psi_0(f,t) = \sum_{i=1}^{n} \sum_{j=1}^{n} U_i(f) U_j(f)$$

24. $$H(t) = \int_{-\infty}^{\infty} \psi_0(f,t)\, df$$

25. $$H(t) = \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{a_i a_j}{2\sqrt{\pi}\sigma} e^{-\pi^2\sigma^2\left(b_j - b_i\right)^2}$$

26. $$\Omega\left(t_1, t_2\right) = \int_{-\infty}^{\infty} \left| \psi_1\left(f, t_2\right) - \psi_1\left(f, t_1\right) \right| \, df$$

27. $$\overrightarrow{A}\left(t_x\right) = \left\{a_{1,x}, a_{2,x}, \ldots, a_{n,x}\right\}$$

28. $$\overrightarrow{B}\left(t_x\right) = \left\{b_{1,x}, b_{2,x}, \ldots, b_{n,x}\right\}$$

29. $$U_{i,x}(f) = a_{i,x} e^{-2\pi^2 \sigma^2 \left(f - b_{i,x}\right)^2}$$

30. $$\psi_1\left(f, t_x\right) = \sum_{g=1}^{\infty} \sum_{h=g}^{\infty} \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{\text{If}[\gcd(g,h) = 1,1,0]}{\sqrt{gh}} U_{i,x}(gf) U_{j,x}(hf)$$