

Tonality Equations

$$1. F(f) = \int_{-\infty}^{\infty} P(t)e^{-2\pi i t f} dt$$

$$2. G(f, t) = \gamma(f) \int_{-\infty}^{\infty} \alpha(u, t) P(u) e^{-2\pi i u f} du$$

$$3. \alpha(u, t) = e^{-\frac{(u-t)^2}{2\sigma^2}}$$

$$4. \tau(f, t) = \frac{2}{\pi\sigma^2} \sum_{g=1}^{\infty} \sum_{h=1}^{\infty} \frac{\mathbb{I}[\gcd(g, h) = 1, 1, 0]}{\sqrt{gh}} G\left(gf, \frac{t}{g}\right) G\left(hf, \frac{t}{h}\right)^*$$

$$5. P(t) = \sum_{i=1}^n a_i \cos(2\pi b_i t + c_i)$$

$$6. G(f, t) = \sigma \sqrt{\frac{\pi}{2}} \sum_{i=1}^n U_i(f) (V_i(f, t) - iW_i(f, t))$$

$$7. U_i(f) = a_i e^{-2\pi^2 \sigma^2 (f - b_i)^2}$$

$$8. V_i(f, t) = \cos \left(2\pi t (f - b_i) - c_i \right)$$

$$9. W_i(f, t) = \sin \left(2\pi t (f - b_i) - c_i \right)$$

$$10. \tau(f, t) = \sum_{g=1}^{\infty} \sum_{h=1}^{\infty} \sum_{i=1}^n \sum_{j=1}^n \frac{\mathbb{1}[\gcd(g, h) = 1, 1, 0]}{\sqrt{gh}} U_i(gf) U_j(hf) \left(X_{g,h,i,j}(t) - i Y_{g,h,i,j}(t) \right)$$

$$11. X_{g,h,i,j}(t) = \cos \left(2\pi t \left(\frac{b_j}{h} - \frac{b_i}{g} \right) + c_j - c_i \right)$$

$$12. Y_{g,h,i,j}(t) = \sin \left(2\pi t \left(\frac{b_j}{h} - \frac{b_i}{g} \right) + c_j - c_i \right)$$

$$13. s = \log_2 \left(\frac{b}{b_0} \right)$$

$$14. b = b_0 2^{s/12}$$

$$15. \psi_1(f, t) = \sqrt{\int_{-\infty}^{\infty} \frac{\alpha(u, t)}{\sigma \sqrt{2\pi}} \tau(f, u) \tau(f, u)^* du}$$

$$16. \psi_2(f, t) = \sqrt{\int_{-\infty}^{\infty} \frac{\alpha(u, t)}{\sigma \sqrt{2\pi}} \frac{\partial \tau(f, u)}{\partial u} \left(\frac{\partial \tau(f, u)}{\partial u} \right)^* du}$$

$$17. \psi_1(f, t) = \sum_{g=1}^{\infty} \sum_{h=1}^{\infty} \sum_{i=1}^n \sum_{j=1}^n \frac{\text{lf}[\text{gcd}(g, h) = 1, 1, 0]}{\sqrt{gh}} U_i(gf) U_j(hf)$$

$$18. \psi_2(f, t) = 2\pi \sum_{g=1}^{\infty} \sum_{h=1}^{\infty} \sum_{i=1}^n \sum_{j=1}^n \frac{\text{lf}[\text{gcd}(g, h) = 1, 1, 0]}{\sqrt{gh}} \left| \frac{b_j}{h} - \frac{b_i}{g} \right| U_i(gf) U_j(hf)$$

$$19. \eta_1(t) = \int_{-\infty}^{\infty} \psi_1(f, t)^2 df$$

$$20. \eta_2(t) = \int_{-\infty}^{\infty} \psi_2(f, t) df$$

$$21. \Omega(t_1, t_2) = \int_{-\infty}^{\infty} \left| \psi_1(f, t_2) - \psi_1(f, t_1) \right| df$$

$$22. \vec{A}(t_x) = \{a_{1,x}, a_{2,x}, \dots, a_{n,x}\}$$

$$23. \vec{B}(t_x) = \{b_{1,x}, b_{2,x}, \dots, b_{n,x}\}$$

$$24. U_{i,x}(f) = a_{i,x} e^{-2\pi^2 \sigma^2 (f - b_{i,x})^2}$$

$$25. \psi_1(f, t_x) = \sum_{g=1}^{\infty} \sum_{h=1}^{\infty} \sum_{i=1}^n \sum_{j=1}^n \frac{\text{lf}[\text{gcd}(g, h) = 1, 1, 0]}{\sqrt{gh}} U_{i,x}(gf) U_{j,x}(hf)$$