

# Triad Tuning

Most modern keyboard instruments and electronic tuners are tuned to what is called “equal temperament.” This means that each half step is exactly equal to one twelfth of an octave. This is perhaps the best compromise for instruments (like pianos and organs) that cannot adjust their pitch on the fly. However, instruments which are capable of adjusting their pitch on the fly are capable of what is called “just intonation.” Just intonation is a tuning system where we attempt to align each of the pitches of a chord to a partial in the harmonic series. This will result in a harmony that sounds more stable, rich, and intense. However, tuning justly requires some harmonic analysis and careful listening. The chart below shows us how to justly tune various common triads.

**Lower and Upper ET interval** lists the size of the two component intervals in a given equally tempered (ET) triad in semitones measured from the middle and upper pitch to the lowest pitch in a given triad.

**Lower, Middle, and Upper partial** shows to which partials of a harmonic series to which we might tune each pitch in our ET triad to achieve just intonation.

**Lower and Upper Just interval** shows how to retune the Lower and Upper ET intervals to tune the triad justly to this combination of partials.

**Lower and Upper Adjustment** measures how far to adjust the Lower and Upper ET intervals in cents (1 cent is 1/100th of a semitone) to achieve this just intonation.

## Just Intonation for Common Triads

Lower ET interval (semitones)	Upper ET interval (semitones)	Lower partial	Middle partial	Upper partial	Lower just interval (semitones)	Upper just interval (semitones)	Lower adjustment (cents)	Upper adjustment (cents)
1	2	18	19	20	0.936	1.824	-6.40	-17.60
1	2	17	18	19	0.990	1.926	-1.05	-7.44
1	2	16	17	18	1.050	2.039	4.96	3.91
1	2	15	16	17	1.117	2.167	11.73	16.69
1	2	14	15	16	1.194	2.312	19.44	31.17
1	2	13	14	15	1.283	2.477	28.30	47.74
1	3	17	18	20	0.990	2.814	-1.05	-18.64
1	3	16	17	19	1.050	2.975	4.96	-2.49
1	3	15	16	18	1.117	3.156	11.73	15.64
1	3	14	15	17	1.194	3.361	19.44	36.13
1	3	12	13	14	1.386	2.669	38.57	-33.13

Lower ET interval (semitones)	Upper ET interval (semitones)	Lower partial	Middle partial	Upper partial	Lower just interval (semitones)	Upper just interval (semitones)	Lower adjustment (cents)	Upper adjustment (cents)
1	4	16	17	20	1.050	3.863	4.96	-13.69
1	4	15	16	19	1.117	4.092	11.73	9.24
1	4	14	15	18	1.194	4.351	19.44	35.08
1	4	13	14	16	1.283	3.595	28.30	-40.53
1	4	12	13	15	1.386	3.863	38.57	-13.69
1	5	15	16	20	1.117	4.980	11.73	-1.96
1	5	14	15	19	1.194	5.287	19.44	28.69
1	5	13	14	17	1.283	4.644	28.30	-35.57
1	5	12	13	16	1.386	4.980	38.57	-1.96
1	6	14	15	20	1.194	6.175	19.44	17.49
1	6	13	14	18	1.283	5.634	28.30	-36.62
1	6	12	13	17	1.386	6.030	38.57	3.00
1	7	13	14	19	1.283	6.570	28.30	-43.01
1	7	13	14	20	1.283	7.458	28.30	45.79
1	7	12	13	18	1.386	7.020	38.57	1.96
1	8	12	13	19	1.386	7.956	38.57	-4.44
1	9	12	13	20	1.386	8.844	38.57	-15.64
2	3	11	12	13	1.506	2.892	-49.36	-10.79
2	3	10	11	12	1.650	3.156	-35.00	15.64
2	3	9	10	11	1.824	3.474	-17.60	47.41
2	3	17	19	20	1.926	2.814	-7.44	-18.64
2	3	16	18	19	2.039	2.975	3.91	-2.49
2	3	15	17	18	2.167	3.156	16.69	15.64
2	3	14	16	17	2.312	3.361	31.17	36.13
2	4	11	12	14	1.506	4.175	-49.36	17.51
2	4	8	9	10	2.039	3.863	3.91	-13.69
2	4	15	17	19	2.167	4.092	16.69	9.24
2	4	7	8	9	2.312	4.351	31.17	35.08
2	4	13	15	16	2.477	3.595	47.74	-40.53
2	5	11	12	15	1.506	5.370	-49.36	36.95
2	5	10	11	13	1.650	4.542	-35.00	-45.79
2	5	9	10	12	1.824	4.980	-17.60	-1.96
2	5	15	17	20	2.167	4.980	16.69	-1.96

Lower ET interval (semitones)	Upper ET interval (semitones)	Lower partial	Middle partial	Upper partial	Lower just interval (semitones)	Upper just interval (semitones)	Lower adjustment (cents)	Upper adjustment (cents)
2	5	14	16	19	2.312	5.287	31.17	28.69
2	5	13	15	17	2.477	4.644	47.74	-35.57
2	6	11	12	16	1.506	6.487	-49.36	48.68
2	6	10	11	14	1.650	5.825	-35.00	-17.49
2	6	9	10	13	1.824	6.366	-17.60	36.62
2	6	8	9	11	2.039	5.513	3.91	-48.68
2	6	7	8	10	2.312	6.175	31.17	17.49
2	6	13	15	18	2.477	5.634	47.74	-36.62
2	7	10	11	15	1.650	7.020	-35.00	1.96
<b>2</b>	<b>7</b>	<b>8</b>	<b>9</b>	<b>12</b>	<b>2.039</b>	<b>7.020</b>	<b>3.91</b>	<b>1.96</b>
2	7	13	15	19	2.477	6.570	47.74	-43.01
2	7	13	15	20	2.477	7.458	47.74	45.79
2	8	11	12	17	1.506	7.536	-49.36	-46.36
2	8	10	11	16	1.650	8.137	-35.00	13.69
2	8	9	10	14	1.824	7.649	-17.60	-35.08
2	8	8	9	13	2.039	8.405	3.91	40.53
2	8	7	8	11	2.312	7.825	31.17	-17.51
2	9	11	12	18	1.506	8.526	-49.36	-47.41
2	9	11	12	19	1.506	9.462	-49.36	46.20
2	9	10	11	17	1.650	9.186	-35.00	18.64
2	9	9	10	15	1.824	8.844	-17.60	-15.64
2	9	7	8	12	2.312	9.331	31.17	33.13
2	10	11	12	20	1.506	10.350	-49.36	35.00
2	10	10	11	18	1.650	10.176	-35.00	17.60
2	10	9	10	16	1.824	9.961	-17.60	-3.91
2	10	8	9	14	2.039	9.688	3.91	-31.17
2	11	10	11	19	1.650	11.112	-35.00	11.20
2	11	9	10	17	1.824	11.010	-17.60	1.05
2	11	8	9	15	2.039	10.883	3.91	-11.73
2	11	7	8	13	2.312	10.717	31.17	-28.30
3	4	12	14	15	2.669	3.863	-33.13	-13.69
3	4	11	13	14	2.892	4.175	-10.79	17.51

Lower ET interval (semitones)	Upper ET interval (semitones)	Lower partial	Middle partial	Upper partial	Lower just interval (semitones)	Upper just interval (semitones)	Lower adjustment (cents)	Upper adjustment (cents)
3	4	16	19	20	2.975	3.863	-2.49	-13.69
3	4	15	18	19	3.156	4.092	15.64	9.24
3	4	14	17	18	3.361	4.351	36.13	35.08
3	5	6	7	8	2.669	4.980	-33.13	-1.96
3	5	11	13	15	2.892	5.370	-10.79	36.95
3	5	10	12	13	3.156	4.542	15.64	-45.79
3	5	15	18	20	3.156	4.980	15.64	-1.96
3	5	14	17	19	3.361	5.287	36.13	28.69
3	5	9	11	12	3.474	4.980	47.41	-1.96
3	6	12	14	17	2.669	6.030	-33.13	3.00
3	6	11	13	16	2.892	6.487	-10.79	48.68
<b>3</b>	<b>6</b>	<b>5</b>	<b>6</b>	<b>7</b>	<b>3.156</b>	<b>5.825</b>	<b>15.64</b>	<b>-17.49</b>
3	6	14	17	20	3.361	6.175	36.13	17.49
3	6	9	11	13	3.474	6.366	47.41	36.62
3	7	6	7	9	2.669	7.020	-33.13	1.96
<b>3</b>	<b>7</b>	<b>10</b>	<b>12</b>	<b>15</b>	<b>3.156</b>	<b>7.020</b>	<b>15.64</b>	<b>1.96</b>
3	8	12	14	19	2.669	7.956	-33.13	-4.44
3	8	11	13	17	2.892	7.536	-10.79	-46.36
3	8	5	6	8	3.156	8.137	15.64	13.69
3	8	9	11	14	3.474	7.649	47.41	-35.08
3	9	6	7	10	2.669	8.844	-33.13	-15.64
3	9	11	13	18	2.892	8.526	-10.79	-47.41
3	9	11	13	19	2.892	9.462	-10.79	46.20
3	9	10	12	17	3.156	9.186	15.64	18.64
3	9	9	11	15	3.474	8.844	47.41	-15.64
3	10	6	7	11	2.669	10.494	-33.13	49.36
3	10	11	13	20	2.892	10.350	-10.79	35.00
3	10	5	6	9	3.156	10.176	15.64	17.60
3	10	9	11	16	3.474	9.961	47.41	-3.91
3	11	10	12	19	3.156	11.112	15.64	11.20
3	11	9	11	17	3.474	11.010	47.41	1.05
4	5	13	16	17	3.595	4.644	-40.53	-35.57

Lower ET interval (semitones)	Upper ET interval (semitones)	Lower partial	Middle partial	Upper partial	Lower just interval (semitones)	Upper just interval (semitones)	Lower adjustment (cents)	Upper adjustment (cents)
4	5	12	15	16	3.863	4.980	-13.69	-1.96
4	5	15	19	20	4.092	4.980	9.24	-1.96
4	5	11	14	15	4.175	5.370	17.51	36.95
4	5	14	18	19	4.351	5.287	35.08	28.69
4	6	13	16	18	3.595	5.634	-40.53	-36.62
4	6	8	10	11	3.863	5.513	-13.69	-48.68
4	6	12	15	17	3.863	6.030	-13.69	3.00
4	6	11	14	16	4.175	6.487	17.51	48.68
4	6	7	9	10	4.351	6.175	35.08	17.49
4	7	13	16	19	3.595	6.570	-40.53	-43.01
4	7	13	16	20	3.595	7.458	-40.53	45.79
<b>4</b>	<b>7</b>	<b>4</b>	<b>5</b>	<b>6</b>	<b>3.863</b>	<b>7.020</b>	<b>-13.69</b>	<b>1.96</b>
4	8	12	15	19	3.863	7.956	-13.69	-4.44
4	8	8	10	13	3.863	8.405	-13.69	40.53
4	8	11	14	17	4.175	7.536	17.51	-46.36
<b>4</b>	<b>8</b>	<b>7</b>	<b>9</b>	<b>11</b>	<b>4.351</b>	<b>7.825</b>	<b>35.08</b>	<b>-17.51</b>
4	9	12	15	20	3.863	8.844	-13.69	-15.64
4	9	11	14	18	4.175	8.526	17.51	-47.41
4	9	11	14	19	4.175	9.462	17.51	46.20
4	9	7	9	12	4.351	9.331	35.08	33.13
4	10	4	5	7	3.863	9.688	-13.69	-31.17
4	10	11	14	20	4.175	10.350	17.51	35.00
4	11	8	10	15	3.863	10.883	-13.69	-11.73
4	11	7	9	13	4.351	10.717	35.08	-28.30
5	6	10	13	14	4.542	5.825	-45.79	-17.49
5	6	13	17	18	4.644	5.634	-35.57	-36.62
5	6	12	16	17	4.980	6.030	-1.96	3.00
5	6	9	12	13	4.980	6.366	-1.96	36.62
5	6	14	19	20	5.287	6.175	28.69	17.49
5	6	11	15	16	5.370	6.487	36.95	48.68
5	7	10	13	15	4.542	7.020	-45.79	1.96
5	7	13	17	19	4.644	6.570	-35.57	-43.01

Lower ET interval (semitones)	Upper ET interval (semitones)	Lower partial	Middle partial	Upper partial	Lower just interval (semitones)	Upper just interval (semitones)	Lower adjustment (cents)	Upper adjustment (cents)
5	7	13	17	20	4.644	7.458	-35.57	45.79
<b>5</b>	<b>7</b>	<b>6</b>	<b>8</b>	<b>9</b>	<b>4.980</b>	<b>7.020</b>	<b>-1.96</b>	<b>1.96</b>
5	8	10	13	16	4.542	8.137	-45.79	13.69
5	8	9	12	14	4.980	7.649	-1.96	-35.08
5	8	12	16	19	4.980	7.956	-1.96	-4.44
5	8	11	15	17	5.370	7.536	36.95	-46.36
5	9	10	13	17	4.542	9.186	-45.79	18.64
5	9	3	4	5	4.980	8.844	-1.96	-15.64
5	9	11	15	18	5.370	8.526	36.95	-47.41
5	9	11	15	19	5.370	9.462	36.95	46.20
5	10	10	13	18	4.542	10.176	-45.79	17.60
5	10	9	12	16	4.980	9.961	-1.96	-3.91
5	10	6	8	11	4.980	10.494	-1.96	49.36
5	10	11	15	20	5.370	10.350	36.95	35.00
5	11	10	13	19	4.542	11.112	-45.79	11.20
5	11	9	12	17	4.980	11.010	-1.96	1.05
6	7	8	11	12	5.513	7.020	-48.68	1.96
6	7	13	18	19	5.634	6.570	-36.62	-43.01
6	7	13	18	20	5.634	7.458	-36.62	45.79
6	7	10	14	15	5.825	7.020	-17.49	1.96
6	7	12	17	18	6.030	7.020	3.00	1.96
6	8	8	11	13	5.513	8.405	-48.68	40.53
6	8	5	7	8	5.825	8.137	-17.49	13.69
6	8	12	17	19	6.030	7.956	3.00	-4.44
6	8	7	10	11	6.175	7.825	17.49	-17.51
6	8	9	13	14	6.366	7.649	36.62	-35.08
6	8	11	16	17	6.487	7.536	48.68	-46.36
6	9	10	14	17	5.825	9.186	-17.49	18.64
6	9	12	17	20	6.030	8.844	3.00	-15.64
6	9	7	10	12	6.175	9.331	17.49	33.13
6	9	9	13	15	6.366	8.844	36.62	-15.64
6	9	11	16	18	6.487	8.526	48.68	-47.41

Lower ET interval (semitones)	Upper ET interval (semitones)	Lower partial	Middle partial	Upper partial	Lower just interval (semitones)	Upper just interval (semitones)	Lower adjustment (cents)	Upper adjustment (cents)
6	9	11	16	19	6.487	9.462	48.68	46.20
6	10	8	11	14	5.513	9.688	-48.68	-31.17
6	10	5	7	9	5.825	10.176	-17.49	17.60
6	10	9	13	16	6.366	9.961	36.62	-3.91
6	10	11	16	20	6.487	10.350	48.68	35.00
6	11	8	11	15	5.513	10.883	-48.68	-11.73
6	11	10	14	19	5.825	11.112	-17.49	11.20
6	11	7	10	13	6.175	10.717	17.49	-28.30
6	11	9	13	17	6.366	11.010	36.62	1.05
7	7	13	19	20	6.570	7.458	-43.01	45.79
7	8	12	18	19	7.020	7.956	1.96	-4.44
7	8	10	15	16	7.020	8.137	1.96	13.69
7	8	8	12	13	7.020	8.405	1.96	40.53
7	9	6	9	10	7.020	8.844	1.96	-15.64
7	9	10	15	17	7.020	9.186	1.96	18.64
7	10	4	6	7	7.020	9.688	1.96	-31.17
7	10	10	15	18	7.020	10.176	1.96	17.60
7	10	6	9	11	7.020	10.494	1.96	49.36
7	11	8	12	15	7.020	10.883	1.96	-11.73
7	11	10	15	19	7.020	11.112	1.96	11.20
8	9	11	17	18	7.536	8.526	-46.36	-47.41
8	9	11	17	19	7.536	9.462	-46.36	46.20
8	9	9	14	15	7.649	8.844	-35.08	-15.64
8	9	7	11	12	7.825	9.331	-17.51	33.13
8	9	12	19	20	7.956	8.844	-4.44	-15.64
8	9	10	16	17	8.137	9.186	13.69	18.64
8	10	11	17	20	7.536	10.350	-46.36	35.00
8	10	9	14	16	7.649	9.961	-35.08	-3.91
8	10	5	8	9	8.137	10.176	13.69	17.60
8	10	8	13	14	8.405	9.688	40.53	-31.17
8	11	9	14	17	7.649	11.010	-35.08	1.05
8	11	7	11	13	7.825	10.717	-17.51	-28.30
8	11	10	16	19	8.137	11.112	13.69	11.20

Lower ET interval (semitones)	Upper ET interval (semitones)	Lower partial	Middle partial	Upper partial	Lower just interval (semitones)	Upper just interval (semitones)	Lower adjustment (cents)	Upper adjustment (cents)
8	11	8	13	15	8.405	10.883	40.53	-11.73
9	9	11	18	19	8.526	9.462	-47.41	46.20
9	10	11	18	20	8.526	10.350	-47.41	35.00
9	10	9	15	16	8.844	9.961	-15.64	-3.91
9	10	6	10	11	8.844	10.494	-15.64	49.36
9	10	10	17	18	9.186	10.176	18.64	17.60
9	10	11	19	20	9.462	10.350	46.20	35.00
9	11	9	15	17	8.844	11.010	-15.64	1.05
9	11	10	17	19	9.186	11.112	18.64	11.20
9	11	7	12	13	9.331	10.717	33.13	-28.30
10	11	8	14	15	9.688	10.883	-31.17	-11.73
10	11	9	16	17	9.961	11.010	-3.91	1.05
10	11	10	18	19	10.176	11.112	17.60	11.20